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Investigation of the vibration problem of sandwich heterogeneous orthotropic plates

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Abstract

In this study, the solution of the free vibration problem of sandwich plates consisting of heterogeneous orthotropic layers is presented. First, the Kirchhoff-Love theory (KLT) for homogeneous plates is extended to sandwich plates composed of heterogeneous orthotropic layers. Within the framework of Donnell theory, after establishing the basic relations of multilayer plates, governing equations are derived based on Airy stress and deflection functions. The solution of the governing equations is carried out by the Galerkin method and the analytical expression is found for the linear frequency of sandwich plates consisting of heterogeneous orthotropic layers. Finally, the effects of various factors such as heterogeneity, number of layers and their arrangement on the free vibration frequency of sandwich plates are examined.

Keywords: heterogeneity, sandwich plates, vibration, frequency.
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1. Introduction

Sandwich structural elements, consisting of two surface layers and a core, are frequently used in different areas of contemporary engineering, from satellites to airplanes, from ships to automobiles, from rail vehicles to wind energy systems and bridge construction. The surface layers of sandwich structural elements may differ in thickness and material. The need for high-performance, low-weight structures ensures that sandwich structural elements consisting of heterogeneous materials continue to be in demand [1]. Since plates with various configurations are widely used in engineering structures, it is becoming increasingly important to study their dynamic behavior, especially their vibration behavior. The solution of dynamic problems of sandwich structural elements is associated with some difficulties. One of the most important of these difficulties is the heterogeneity of layers of sandwich structural elements with different properties. These factors makes it necessary in vibration problems of sandwich structural elements to take into account the heterogeneity of materials in the layers and study their influence on the vibration frequency. Although efforts to create analytical models that reflect the reality of the mechanical properties of heterogeneous orthotropic materials are always on the agenda, the number of modeling studies is limited [2, 3].

One of the main goals when designing heterogeneous multilayer or sandwich structural elements is to make their vibration calculations. In most cases, in vibration calculations of structural elements consisting of homogeneous materials, it is seen that their frequencies are much different than in reality. One of the reasons for this phenomenon lies in the heterogeneous anisotropic nature of the layers forming the sandwich structural element. To date, some studies have been carried out on vibrations of multilayer or sandwich heterogeneous plates. Amon those, Fares and Zenkour [4] presented buckling and free vibration of non-homogeneous composite cross-ply laminated plates with various plate theories. Kuo and Shiau [5] studied buckling and vibration of composite laminated plates with variable fiber spacing. Orakdogen et al. [6] presented finite element analysis of functionally graded plates for coupling effect of extension and bending**.** Zerin [7] studied the vibration problem of laminated nonhomogeneous orthotropic shells. Dozio [8] investigated vibration behaviors of sandwich plates with FGM core via variable-kinematic 2-D Ritz models. Chen et al. [9] investigated vibration and stability of initially stressed sandwich plates with FGM face sheets in thermal environments. Haciyev et al. [10] solved the free bending vibration problem of thin non-homogenous orthotropic rectangular plates resting on viscoelastic foundations. Bacciocchi and Tarantino [11] presented natural frequency analysis of functionally graded orthotropic cross-ply plates based on the finite element method. Bouazza and Zenkour [12] studied vibration of inhomogeneous fibrous laminated plates using an efficient and simple polynomial refined theory. Alnujaie et al. [13] studied buckling and free vibration analysis of multi-directional functionally graded sandwich plates. Li et al. [14] examined free vibration of functionally graded sandwich plates in thermal environments.

In this study, the vibration problem of sandwich plates consisting of heterogeneous layers will be discussed within the framework of classical plate theory. This theory is based on the Kirchhoff-Love theory, which assumes that the normal to the mid-plane remain straight and normal to the mid-plane remains normal before and after deformation and the size does not change.

2. Experiments

Consider a sandwich plate with total thickness h, lengths a and b on the x and y axes, respectively, which consists of heterogeneous orthotropic three layers. The geometry and coordinate system are shown in Figure 1. The origin of the coordinate system $Oxyz$ is in the upper left corner of the reference plane of the sandwich plate, while the x and y axes are on the $z=0$ reference plane, the z axis is perpendicular to the reference plane and directed inwards.

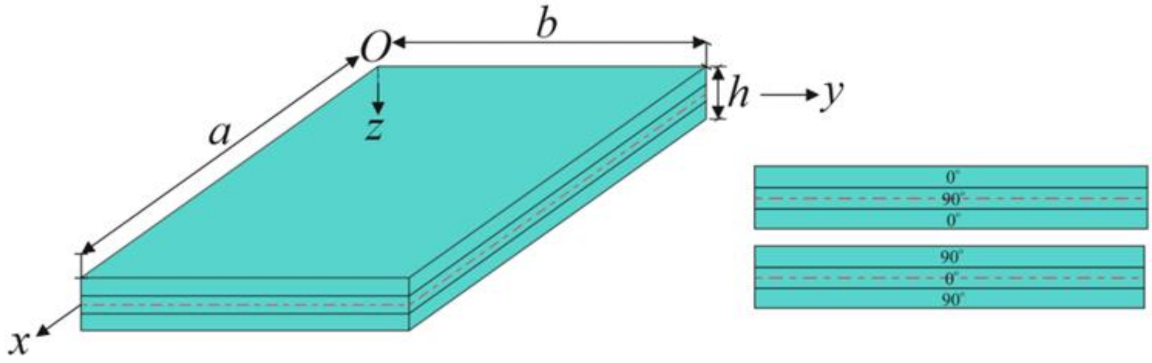

Figure 1. The geometry, coordinate system and cross section of the sandwich plate

It is assumed that the layers of sandwich plate are perfectly bonded to each other, they do not slip and remain elastic during deformation. The displacements in the x, y and z directions are indicated by U, V and W, respectively. Let $\Phi(x,y,t)$ be the Airy stress function for the stress resultants, so that [15],

$$N_{11} = \frac{\partial^2 \Phi}{\partial y^2},\ N_{22} = \frac{\partial^2 \Phi}{\partial x^2},\ \ N_{12} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{1}$$

Where $N_{ij}\,(i,\,j=1,2)$ are forces.

The mechanical properties such as Young moduli $\left(E_{11Z}^{(k)},\ E_{22Z}^{(k)}\right)$, shear modulus $\left(G_{12Z}^{(k)}\right)$ and density $(\rho_Z^{(k)})$ of lamina k^{th} are linear and quadratic functions of the thickness coordinate and defined as follows [2-4,7,10]:

$$\begin{gathered} E_{11Z}^{(k)} = \left(1+\mu z^n\right)E_{11}^{0(k)},\ E_{22Z}^{(k)} = \left(1+\mu z^n\right)E_{22}^{0(k)},\ G_{12Z}^{(k)} = \left(1+\mu_1 z^n\right)G_{12}^{0(k)}, \\ \rho_Z = \left(1+\mu z^n\right)\rho^{0(\mathrm{k})},\ Z = z/h,\ n=1,2 \end{gathered} \tag{2}$$

where the symbols with “0” in the superscript indicate the mechanical properties of the homogeneous orthotropic material, μ indicates the heterogeneity parameter for the elasticity moduli and density in the layer of sandwich plates, which characterizes its variation depending on the Z and $\mu \in [0,1]$. In addition, $\mu=0$ case indicates homogeneous orthotropic material.

3. Basic equations

Based on the KLT, the motion and strain compatibility equations for the layered plates composed of heterogeneous layers could be expressed as [15]:

$$\frac{\partial^2 M_{11}}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_{22}}{\partial y^2} = \bar{\rho}\,\frac{\partial^2 w}{\partial t^2} \tag{3}$$

$$\frac{\partial^2 e_{11}^0}{\partial y^2} + \frac{\partial^2 e_{22}^0}{\partial x^2} - \frac{\partial^2 \gamma_{12}^0}{\partial x \partial y} = 0 \tag{4}$$

where t is the time, M_{ij} are forces, $e_{ii}^0, \gamma_{12}\,(i=1,2)$ are strains in the reference plane, and $\bar{\rho}$ is the density parameter of sandwich plates are defined, as follows [3, 16]:

$$\left(N_{ij}, M_{ij}\right) = \sum_{k=1}^{3}\int_{z_{k-1}}^{z_k} \sigma_{ij}^{(k)}(1,z)\mathrm{d}z,\ (i,\,j=1,2,6),\ \ \bar{\rho} = \sum_{k=1}^{3}\int_{z_{k-1}}^{z_k} \rho_t^{(k)}\mathrm{d}z \tag{5}$$

in which

$$-\frac{h}{2} + \frac{(k-1)h}{N} \le z \le -\frac{h}{2} + \frac{kh}{N} \tag{6}$$

The relationships between stresses $\sigma_{ij}^{(k)}(i,\,j=1,2,k=1,2,3)$, and strains e_{ij} and γ_{12} for the k^{th} lamina contained in sandwich plates composed of heterogeneous layers can be written as follows [2,7],

$$\begin{pmatrix}\sigma_{11}^{(k)}\\ \sigma_{22}^{(k)}\\ \sigma_{12}^{(k)}\end{pmatrix}=\begin{bmatrix}\bar{E}_{11Z}^{(k)} & \bar{E}_{12Z}^{(k)} & 0 & 0 & 0\\ \bar{E}_{21Z}^{(k)} & \bar{E}_{22Z}^{(k)} & 0 & 0 & 0\\ 0 & 0 & \bar{E}_{66Z}^{(k)} & 0 & 0\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ \gamma_{12}\end{bmatrix} \tag{7}$$

where

$$\bar{E}_{11Z}^{(k)}=\frac{E_{11Z}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\ \bar{E}_{12Z}^{(k)}=\frac{\nu_{21}^{(k)}E_{11Z}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}=\frac{\nu_{12}^{(k)}E_{22Z}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}=\bar{E}_{21Z}^{(k)},\bar{E}_{22Z}^{(k)}=\frac{E_{22Z}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\ \bar{E}_{66Z}^{(k)}=G_{12Z}^{(k)} \tag{8}$$

in which $E_{iiZ}^{(k)},\ G_{12Z}^{(k)}(i=1,2,6)$ are Young and shear moduli of heterogeneous orthotropic materials in the lamina k^{th}, $\nu_{12}^{(k)}$ and $\nu_{21}^{(k)}$ are Poisson ratios and are considered constant since the effect of heterogeneity according to the thickness coordinate is very small and the following condition is satisfied: $\nu_{21}^{(k)}E_{11}^{0(k)}=\nu_{12}^{(k)}E_{22}^{0(k)}$.

By using the relationships from (1) to (7), the motion and deformation compatibility equations for sandwich heterogeneous plates composed within KLT are obtained as follows:

$$\begin{aligned}&c_{12}\frac{\partial^4\Phi}{\partial x^4}+\left(c_{11}-2c_{31}+c_{22}\right)\frac{\partial^4\Phi}{\partial x^2\partial y^2}+c_{21}\frac{\partial^4\Phi}{\partial y^4}-c_{13}\frac{\partial^4 w}{\partial x^4}\\&-\left(c_{14}+2c_{32}+c_{13}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}-c_{14}\frac{\partial^4 w}{\partial y^4}=\bar{\rho}\frac{\partial^2 w}{\partial t^2}\end{aligned} \tag{9}$$

$$b_{22}\frac{\partial^4\Phi}{\partial x^4}+\left(b_{12}+b_{21}+b_{31}\right)\frac{\partial^4\Phi}{\partial x^2\partial y^2}+b_{11}\frac{\partial^4\Phi}{\partial y^4}-\left(b_{13}-b_{32}+b_{24}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}-b_{14}\frac{\partial^4 w}{\partial y^4}-b_{23}\frac{\partial^4 w}{\partial x^4}=0 \tag{10}$$

where b_{ij}, c_{ij} $(i, j=1,2,..,4)$ are coefficients depending on the material properties of sandwich plates composed of heterogeneous layers.

4. Solution procedure

Since all edges of the sandwich plate are assumed to be simply supported, the solution of (9) and (10) is sought as follows [2, 15]:

$$w=w_1(t)\sin(m_1x)\sin(m_2y),\ \Phi=\phi_1(t)\sin(m_1x)\sin(m_2y) \tag{11}$$

where $w_1(t)$ and $\phi_1(t)$ are time dependent functions, $m_1=\frac{m\pi}{a}$ and $m_2=\frac{n\pi}{b}$ are the wave parameters in which (m, n) is the vibration mode.

Substituting (11) into the set of Eqs (9) and (10), and eliminating the unknown $\phi_1(t)$ from the resulting equations, one gets,

$$\frac{\mathrm{d}^2 w_1}{\mathrm{d}t^2}+\alpha w_1=0 \tag{12}$$

where the following definition applies:

$$\alpha=\frac{1}{\bar{\rho}}\left[-c_{21}m_2^{\ 4}-(c_{11}-2c_{31}+c_{22})m_2^{\ 2}m_1^{\ 2}-c_{12}m_1^{\ 4}\right]\times\frac{b_{23}m_1^{\ 4}+\left(b_{24}+b_{13}-b_{32}\right)m_1^{\ 2}m_2^{\ 2}+b_{14}m_2^{\ 4}}{b_{11}m_2^{\ 4}+(b_{12}+b_{21}+b_{31})m_2^{\ 2}m_1^{\ 2}+b_{22}m_1^{\ 4}}$$

$$+c_{13}m_1^{\ 4}+\left(c_{14}+2c_{32}+c_{23}\right)m_1^{\ 2}m_2^{\ 2}+c_{24}m_2^{\ 4} \tag{13}$$

The dimensionless frequency parameter of sandwich plates composed of heterogeneous orthotropic layers within KLT is defined as:

$$\omega_1=h\sqrt{\alpha\rho_0^{(k)}/E_{11}^{0(k)}} \tag{14}$$

5. Results and discussion

The numerical results are performed for free vibration frequency of sandwich rectangular plates made of heterogeneous orthotropic layers, using Eq. (14). The heterogeneity parameters for elasticity moduli and density are used as $\mu=1$ and is denoted as HT in figure. The $\mu=0$ corresponds to homogeneous case and is denoted as H. The properties of homogeneous orthotropic material are taken from the study of Reddy [16]: $E_{11}^{0(k)}=2.069\times10^{11}$ Pa, $E_{22}^{0(k)}=2.069\times10^{10}$ Pa, $G_{12}^{0(k)}=G_{13}^{0(k)}=6.9\times10^{9}$ Pa, $G_{23}^{0(k)}=4.14\times10^{9}$ Pa, $\rho_0^{(k)}=1950\,\mathrm{kg/m^3}$ and $\nu_{12}^{(k)}=0.3$. For subsequent examples use the following characteristics: *a/b* =0.5,1.0,1.5,2.0, *a/h*=15 and *(m,n)=(1,1)*.

The variations of the ω_1 for the H and HT-sandwich rectangular plates versus the *a/b* are presented in Table 1 and Figure 2. As *a/b* increases, the values of ω_1 for single-layer and sandwich plates increase.

When the *a/b* ratio increases from 0.5 to 2; the effect of the heterogeneous-linear profile on the frequency in (0^o) plate is approximately 4.2% and is independent of the increase a/b. As the a/b ratio increases from 0.5 to 2; while the linear heterogeneity effect on frequency values decreases from 5.8% to 3.12 in the (0^o/90^o/0^o)-array plate, this effect increases from 3.2% to 5.83% in the (90^o/0^o/90^o) -array plate. When the a/b ratio increases from 0.5 to 2; the effect of the heterogeneous-quadratic profile on the frequency in (0^o) plate is approximately 7.2% and is independent of the increase of a/b. As the *a/b* ratio increases from 0.5 to 2; while the effect of the heterogeneous-quadratic profile on the frequency values decreases from 7.47% to 6.44 in the (0^o/90^o/0^o)-array plate, this effect increases from 6.4% to 7.42% in the (90^o/0^o/90^o) -array plate.

The effects of layer arrangement on the frequency of sandwich plates vary. For example, while the effects of the homogeneous (0^o/90^o/0^o)-arrangement on the frequency compared to the (0^o)-single layer plate are 1.52%, 0%, 3.39% and 7.16%, respectively, those effects in the plate with the (90^o/0^o/90^o)-arrangement are 52.14%, 0%, 67.82% and 120.45%, respectively, as a/b=0.5, 1.0, 1.5 and 2, respectively. In the heterogeneous-linear case, the effects of the (0^o/90^o/0^o) arrangement on the frequency compared to the (0^o)-single layer plate are 3.12%, 1.01%, 3.54% and 8.41%, respectively, while these effects in the (90^o/0^o/90^o)-sequenced plate are 51.6%, 1.01%, 65.3% and 116.8%, as a/b=0.5, 1.0, 1.5 and 2, respectively. It can be seen that in the heterogeneous-quadratic case, the effects of the (0^o/90^o/0^o)-arrangement on the frequency compared to the (0^o)-single layer plate are 3.12%, 1.01%, 3.54% and 8.41%, respectively, while these effects are in the (90^o/0^o/90^o)-aligned plate are 51.6%, 1.01%, 65.3% and 116.8%, when a/b=0.5, 1.0, 1.5 and 2, respectively.

a / b	Heterogeneous -linear profile					
	(0°)		(0°/90°/0°)		(90°/0°/90°)	
	H	NH	H	NH	H	NH
0.5	1.306	1.250	1.286	1.211	0.625	0.605
1.0	1.445	1.384	1.445	1.370	1.445	1.370
1.5	1.771	1.695	1.831	1.755	2.972	2.802
2.0	2.334	2.235	2.501	2.423	5.146	4.846
	Heterogeneous -quadratic profile					
a / b	(0°)		(0°/90°/0°)		(90°/0°/90°)	
	H	NH	H	NH	H	NH
0.5	1.306	1.400	1.286	1.382	0.625	0.665
1.0	1.445	1.550	1.445	1.55	1.445	1.550
1.5	1.771	1.899	1.831	1.957	2.972	1.191
2.0	2.334	2.503	2.501	2.662	5.146	5.528

Table 1. Variation of the ω_1 for the homogenous and heterogeneous linear and quadratic profiled plates versus the *a /b*

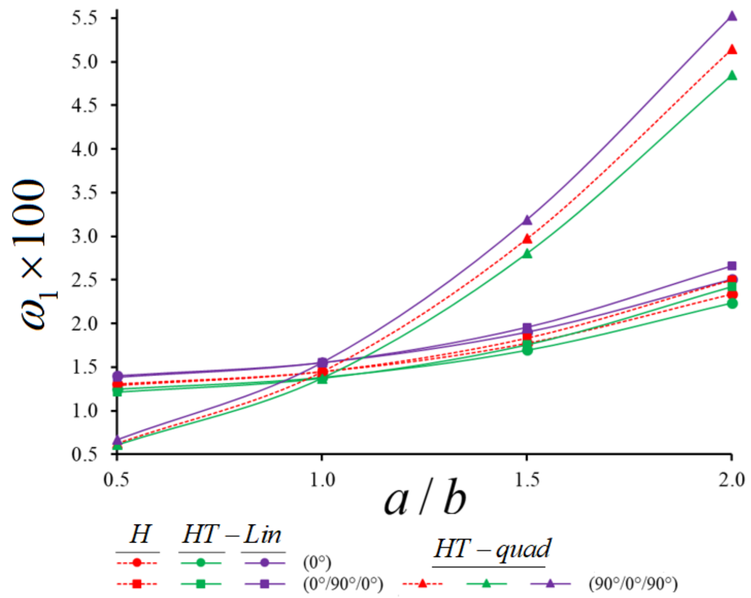

Figure 2. Variation of the ω_1 for the homogenous and heterogeneous linear and quadratic profiled plates versus the *a/b*

5. Conclusion

In this study, the free vibration problem of sandwich plates consisting of heterogeneous orthotropic layers is solved within the framework of Kirchhoff-Love theory (KLT). Within the framework of Donnell type plate theory, the basic relationships of sandwich plates are established and the equation of motion is derived. By applying the Galerkin method, the analytical expression for the linear frequency of sandwich plates consisting of heterogeneous orthotropic layers is obtained. Finally, the importance of the effects of various factors such as inhomogeneity, number and arrangement of layers on the free vibration frequency of square and rectangular plates is examined.

References

1. J.R. Vinson, Applied Mechanics Review **54**(3) (2001) 201.
2. A.H. Sofiyev, E. Schnack, Acta Mechanica **162**(1-4) (2003) 29.
3. Y. Ootao, Y. Tanigawa, Composite Structures **80**(1) (2007) 10.

4. M.E. Fares, A.M. Zenkour, Composite Structures **44**(4) (1999) 279.
5. S.Y. Kuo, L.C. Shiau, Composite Structures **90**(2) (2009) 196.
6. E. Orakdögen, S. Küçükarslan, A. Sofiyev, M.H. Omurtag, Meccanica **45(**1) (2010) 63.
7. Z. Zerin, Meccanica **48**(7) (2013) 1557.
8. L. Dozio, Composite Structures **96** (2013) 561.
9. C.S. Chen, F.H. Liu, W.R. Chen, Steel and Composite Structures **23**(3) (2017) 251.
10. V.C. Haciyev, A.H. Sofiyev, N. Kuruoglu, Mechanics of Advanced Materials and Structures **26**(10) (2019) 886.
11. M. Bacciocchi, A.M. Tarantino, Mathematical and Computational Applications **24**(2) (2019) 52.
12. M. Bouazza, A.M. Zenkour, Journal of Computational Applied Mechanics **52**(2) (2021) 233.
13. A. Alnujaie, A.S. Sayyad, L. Hadji, A. Tounsi, Structural Engineering and Mechanics **84**(6) (2022) 813.
14. D.D. Li, Z.B. Deng, G.P. Chen, International Journal of Mechanical System Dynamics **3**(1) (2023) 39.
15. A.S. Volmir, The nonlinear dynamics of plates and shells. Science Edition, Moscow (1972) 543p.
16. J.N. Reddy, Mechanics of Laminated Composite Plates and Shells: Theory and Analysis. 2nd ed. CRC Press, New-York (2004) 858p.